Spontaneous current generation in the gapless 2SC phase[*]

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It is found that, except chromomagnetic instability, the gapless 2SC phase also exhibits a paramagnetic response to the perturbation of an external color neutral baryon current. The spontaneously generated baryon current driven by the mismatch is equivalent to the one-plane wave LOFF state. We describe the 2SC phase in the nonlinear realization framework, and show that each instability indicates the spontaneous generation of the corresponding pseudo Nambu-Golstone current. We show this Nambu-Goldstone currents generation state covers the gluon phase as well as the one-plane wave LOFF state. We further point out that, when charge neutrality condition is required, there exists a narrow unstable LOFF (Us-LOFF) window, where not only off-diagonal gluons but the diagonal 8-th gluon cannot avoid the magnetic instability. We discuss that the diagonal magnetic instability in this Us-LOFF window cannot be cured by off-diagonal gluon condensate in color superconducting phase, and it will also show up in some constrained Abelian asymmetric superfluid/superconducting system.

I. INTRODUCTION

Sufficiently cold and dense baryonic matter is a color superconductor, such a state of matter may exist in the central region of compact stars. For this reason, the topic of color superconductivity stirred a lot of interest in recent years [1, 2, 3, 4]. (For reviews on color superconductivity see, for example, Ref. [5].)

To form bulk matter inside compact stars, the charge neutrality condition as well as β equilibrium are required [6, 7, 8]. This induces mismatch between the Fermi surfaces of the pairing quarks. It is clear that the Cooper pairing will be eventually destroyed with the increase of mismatch. However, there are no certain answer on the true ground state of neutral cold-dense quark matter with a moderate mismatch.

Without the constraint from the charge neutrality condition, the system may exhibit a first order phase transition from the color superconducting phase to the normal phase when the mismatch increases [9]. It was also found that the system can experience a spatial non-uniform LOFF (Larkin-Ovchinnikov-Fudde-Ferrell) state [10, 11] in a certain window of moderate mismatch.

The charge neutrality condition plays an essential role in determining the ground state of the neutral system. If the charge neutrality condition is satisfied globally, and also if the surface tension is small, the mixed phase will be favored [12]. It is difficult to precisely calculate the surface tension in the mixed phase, thus in the following, we would like to focus on the homogeneous phase when the charge neutrality condition is required locally.

It was found that homogeneous neutral cold-dense quark matter can be in the gapless 2SC (g2SC) phase [13] or gapless CFL (gCFL) phase [14], depending on the fla-

vor structure of the system. The gapless state resembles the unstable Sarma state [15, 16]. However, under a natural charge neutrality condition, i.e., only neutral matter can exist, the gapless phase is indeed a thermal stable state as shown in [13, 14]. The existence of thermal stable gapless color superconducting phases was confirmed in Refs. [17] and generalized to finite temperatures in Refs. [18]. Recent results based on more careful numerical calculations show that the g2SC and gCFL phases can exist at moderate baryon density in the color superconducting phase diagram [19] .

One of the most important properties of an ordinary superconductor is the Meissner effect, i.e., the superconductor expels the magnetic field [20]. In ideal color superconducting phases, e.g., in the 2SC and CFL phases, the gauge bosons connected with the broken generators obtain masses, which indicates the Meissner screening effect [21]. The Meissner effect can be understood using the standard Anderson-Higgs mechanism. Unexpectedly, it was found that in the g2SC phase, the Meissner screening masses for five gluons corresponding to broken generators of $SU(3)_c$ become imaginary, which indicates a type of chromomagnetic instability in the g2SC phase [22, 23]. The calculations in the gCFL phase show the same type of chromomagnetic instability [24]. Remembering the discovery of superfluidity density instability [25] in the gapless interior-gap state [26], it seems that the instability is a inherent property of gapless phases, though the result in Ref. [27] shows that there is no chromomangetic instability near the critical temperature.

The chromomagnetic instability in the gapless phase still remains as a puzzle. By observing that, the 8-th gluon's chromomagnetic instability is related to the instability with respect to a virtual net momentum of diquark pair, Giannakis and Ren suggested that a LOFF state might be the true ground state [28]. Their further calculations show that there is no chromomagnetic instability in a narrow LOFF window when the local stability condition is satisfied [29, 30]. Latter on, it was pointed

out in Ref. [31] that a charge neutral LOFF state cannot cure the instability of off-diagonal 4-7th gluons, while a gluon condensate state [32] can do the job. We will point out that, when charge neutrality condition is required, there exists another narrow unstable LOFF window, not only off-diagonal gluons but the diagonal 8-th gluon cannot avoid the magnetic instability.

This paper is organized as follows. In Sec. II, we describe the framework of the gauged SU(2) Nambu–Jona-Lasinio(NJL) model in β-equilibrium. In Sec. III, we probe the g2SC phase using different external fields, and we find that, except chromomagnetic instability, the g2SC phase is also unstable with respect to a color neutral baryon current. Then in Sec. IV, we propose that a baryon current can be spontaneously generated in the ground state due to the mismatch, and this baryon current generation state resembles the one-plane wave LOFF state. We further analyze the instabilities of the LOFF state under the requirement of charge neutrality condition and point out that there exists a narrow LOFF window, where not only off-diagonal gluons but the diagonal 8-th gluon, cannot avoid the magnetic instability. Sec. V gives a general Nambu-Goldstone currents generation description in the non-linearization framework. At the end, we give the discussion and summary in Sec. VI.

II. THE GAUGED SU(2) NAMBU–JONA-LASINIO (GNJL) MODEL

We take the gauged form of the extended Nambu–Jona-Lasinio model [33], the Lagrangian density has the form of

$$\mathcal{L} = \bar{q}(i\slashed{D} + \hat{\mu}\gamma^0)q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] + G_D[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)], \quad (1)$$

with $D_\mu \equiv \partial_\mu - igA_\mu^a T^a - ieA_\mu Q$. Here A_μ^a and A_μ are gluon fields and photon field, T^a and Q are the generators of $SU(3)_c$ and $U(1)_{EM}$ gauge groups, respectively. Please note that we regard all the gauge fields as external fields, which are weakly interacting with the system. The property of the color superconducting phase characterized by the diquark gap parameter is determined by the unkown nonperturbative gluon fields, which has been simply replaced by the four-fermion interaction in the NJL model. While, the external gluon fields do not contribute to the properties of the system. Therefore, we do not have the contribution to the Lagrangian density from gauge field part $\mathcal{L}_g$ as introduced in [32]. (In Sec. V, by using the non-linear realization in the gNJL model, we will derive one Nambu-Goldstone currents state, which is equivalent to the so-called gluon-condensate state.)

In the Lagrangian density Eq. (1), $q^C = C\bar{q}^T$, $\bar{q}^C = q^T C$ are charge-conjugate spinors, $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (the superscript T denotes the transposition operation). The quark field $q \equiv q_{i\alpha}$ with $i = u, d$ and $\alpha = r, g, b$ is a flavor doublet and

color triplet, as well as a four-component Dirac spinor, $\tau = (\tau^1, \tau^2, \tau^3)$ are Pauli matrices in the flavor space, where τ^2 is antisymmetric, and $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$, $(\epsilon^b)^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are totally antisymmetric tensors in the flavor and color spaces. $\hat{\mu}$ is the matrix of chemical potentials in the color and flavor space. In β-equilibrium, the matrix of chemical potentials in the color-flavor space $\hat{\mu}$ is given in terms of the quark chemical potential μ, the chemical potential for the electrical charge μ_e and the color chemical potential μ_8,

$$\mu_{ij}^{\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij})\delta^{\alpha\beta} + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}(T_8)^{\alpha\beta}. \quad (2)$$

G_S and G_D are the quark-antiquark coupling constant and the diquark coupling constant, respectively. In the following, we only focus on the color superconducting phase, where $< \bar{q}q >= 0$ and $< \bar{q}\gamma^5\tau q >= 0$.

After bosonization, one obtains the linearized version of the model for the 2-flavor superconducting phase,

$$\begin{aligned}\mathcal{L}_{2SC} &= \bar{q}(i\slashed{D} + \hat{\mu}\gamma^0)q - \frac{\Delta^{*b}\Delta^b}{4G_D} \\ &- \frac{1}{2}\Delta^{*b}(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q) - \frac{1}{2}\Delta^b(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C) \quad (3)\end{aligned}$$

with the bosonic fields

$$\Delta^b \sim i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q, \quad \Delta^{*b} \sim i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C. \quad (4)$$

In the Nambu-Gor'kov space,

$$\Psi = \begin{pmatrix} q \\ q^C \end{pmatrix}, \quad (5)$$

the inverse of the quark propagator is defined as

$$[\mathcal{S}(P)]^{-1} = \begin{pmatrix} [G_0^+(P)]^{-1} & \Delta^- \\ \Delta^+ & [G_0^-(P)]^{-1} \end{pmatrix}, \quad (6)$$

with the off-diagonal elements

$$\Delta^- \equiv -i\epsilon^b\varepsilon\gamma_5\Delta, \qquad \Delta^+ \equiv -i\epsilon^b\varepsilon\gamma_5\Delta^*, \quad (7)$$

and the free quark propagators $G_0^\pm(P)$ taking the form of

$$[G_0^\pm(P)]^{-1} = \gamma^0(p_0 \pm \hat{\mu}) - \vec{\gamma} \cdot \vec{p}. \quad (8)$$

The 4-momenta are denoted by capital letters, e.g., $P = (p_0, \vec{p})$. We have assumed the quarks are massless in dense quark matter, and the external gluon fields do not contribute to the quark self-energy.

The explicit form of the functions $G_I^\pm$ and $\Xi_{IJ}^\pm$ reads

$$G_1^\pm = \frac{(k_0 - E_{dg}^\pm)\gamma^0\tilde{\Lambda}_k^+}{(k_0 \mp \delta\mu)^2 - E_\Delta^{\pm 2}} + \frac{(k_0 + E_{dg}^\mp)\gamma^0\tilde{\Lambda}_k^-}{(k_0 \mp \delta\mu)^2 - E_\Delta^{\mp 2}},$$

$$G_2^\pm = \frac{(k_0 - E_{ur}^\pm)\gamma^0\tilde{\Lambda}_k^+}{(k_0 \pm \delta\mu)^2 - E_\Delta^{\pm 2}} + \frac{(k_0 + E_{ur}^\mp)\gamma^0\tilde{\Lambda}_k^-}{(k_0 \pm \delta\mu)^2 - E_\Delta^{\mp 2}},$$

$$G_3^\pm = \frac{1}{k_0 + E_{bu}^\pm}\gamma^0\tilde{\Lambda}_k^+ + \frac{1}{k_0 - E_{bu}^\mp}\gamma^0\tilde{\Lambda}_k^-,$$

$$G_4^\pm = \frac{1}{k_0 + E_{bd}^\pm}\gamma^0\tilde{\Lambda}_k^+ + \frac{1}{k_0 - E_{bd}^\mp}\gamma^0\tilde{\Lambda}_k^-,$$

with $E_{i\alpha}^{\pm} \equiv E_k \pm \mu_{i\alpha}$ and

$$
\Xi_{12}^{\pm} = \left(\frac{-i\Delta\gamma^5\tilde{\Lambda}_k^+}{(k_0 \pm \delta\mu)^2 - E_\Delta^{\pm\,2}} + \frac{-i\Delta\gamma^5\tilde{\Lambda}_k^-}{(k_0 \pm \delta\mu)^2 - E_\Delta^{\mp\,2}} \right),
$$

$$
\Xi_{21}^{\pm} = \left(\frac{-i\Delta\gamma^5\tilde{\Lambda}_k^+}{(k_0 \mp \delta\mu)^2 - E_\Delta^{\pm\,2}} + \frac{-i\Delta\gamma^5\tilde{\Lambda}_k^-}{(k_0 \mp \delta\mu)^2 - E_\Delta^{\mp\,2}} \right),
$$

where

$$
\tilde{\Lambda}_k^{\pm} = \frac{1}{2}\left(1 \pm \gamma^0 \frac{\boldsymbol{\gamma} \cdot \mathbf{k} - m}{E_k} \right) \tag{11}
$$

is an alternative set of energy projectors, and the following notation was used:

$$
\begin{aligned}
E_k^{\pm} &\equiv E_k \pm \bar{\mu}, \\
E_{\Delta,k}^{\pm} &\equiv \sqrt{(E_k^{\pm})^2 + \Delta^2}, \\
\bar{\mu} &\equiv \frac{\mu_{ur} + \mu_{dg}}{2} = \frac{\mu_{ug} + \mu_{dr}}{2} = \mu - \frac{\mu_e}{6} + \frac{\mu_8}{3}, \\
\delta\mu &\equiv \frac{\mu_{dg} - \mu_{ur}}{2} = \frac{\mu_{dr} - \mu_{ug}}{2} = \frac{\mu_e}{2}.
\end{aligned}
$$

III. PARAMAGNETIC RESPONSE TO A COLOR NEUTRAL BARYON CURRENT

It is not understood why gapless color superconducting phases exhibit chromomagnetic instability. It sounds quite strange especially in the g2SC phase, where it is the electrical neutrality not the color neutrality playing the essential role. It is a puzzle why the gluons can feel the instability by requiring the electrical neutrality on the system. In order to understand what is really going 'wrong' with the homogeneous g2SC phase, we want to know whether there exists other instabilities except the chromomagnetic instability. For that purpose, we probe the g2SC phase using different external sources, e.g., scalar and vector diquarks, mesons, vector current, and so on. In this paper, we only report the most interesting result regarding the response of the g2SC phase to an external vector current $V^\mu = \bar{\psi}\gamma^\mu\psi$, the time-component and spatial-components of this current correspond to the baryon number density and baryon current, respectively.

From the linear response theory, the induced current and the external vector current is related by the response function $\Pi_V^{\mu\nu}(P)$,

$$
\Pi_V^{\mu\nu}(P) = \frac{1}{2}\frac{T}{V}\sum_K \text{Tr}\left[\hat{\Gamma}_V^\mu \mathcal{S}(K)\hat{\Gamma}_V^\nu \mathcal{S}(K-P) \right]. \tag{13}
$$

The trace here runs over the Nambu-Gorkov, flavor, color and Dirac indices. The explicit form of vertices is $\hat{\Gamma}_V^\mu \equiv \text{diag}(\gamma^\mu, -\gamma^\mu)$.

The explicit expression of the vector current response function can be evaluated directly by using the notations in Ref. [23],

$$
\Pi_V^{\mu\nu}(P) = \tilde{\Pi}_V^{\mu\nu}(P) + \Pi_{V,b}^{\mu\nu}(P), \tag{14}
$$

$$
\begin{aligned}
\tilde{\Pi}_V^{\mu\nu}(P) = \frac{T}{2}\sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3}\text{Tr}_D\Big[\\
\gamma^\mu G_1^+(K)\gamma^\nu G_1^+(K') + \gamma^\mu G_1^-(K)\gamma^\nu G_1^-(K') \\
+ \gamma^\mu G_2^+(K)\gamma^\nu G_2^+(K') + \gamma^\mu G_2^-(K)\gamma^\nu G_2^-(K') \\
- \gamma^\mu \Xi_{12}^-(K)\gamma^\nu \Xi_{21}^+(K') - \gamma^\mu \Xi_{12}^+(K)\gamma^\nu \Xi_{21}^-(K') \\
- \gamma^\mu \Xi_{21}^-(K)\gamma^\nu \Xi_{12}^+(K') - \gamma^\mu \Xi_{21}^+(K)\gamma^\nu \Xi_{12}^-(K') \Big],
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{V,b}^{\mu\nu}(P) = \frac{T}{2}\sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3}\text{Tr}_D\Big[\\
\gamma^\mu G_3^+(K)\gamma^\nu G_3^+(K') + \gamma^\mu G_3^-(K)\gamma^\nu G_3^-(K') \\
+ \gamma^\mu G_4^+(K)\gamma^\nu G_4^+(K') + \gamma^\mu G_4^-(K)\gamma^\nu G_4^-(K') \Big],
\end{aligned}
$$

here the trace is over the Dirac space.

Comparing the explicit expression of $\Pi_V^{\mu\nu}(P)$ with that of the 8-th gluon's self-energy $\Pi_{88}^{\mu\nu}(P)$, i.e., Eq. (55) in Ref. [23], it can be clearly seen that, $\Pi_V^{\mu\nu}(P)$ and $\Pi_{88}^{\mu\nu}(P)$ almost share the same expression, except the coefficients. This can be easily understood, because the color charge and color current carried by the 8-th gluon is proportional to the baryon number and baryon current, respectively. In the static long-wavelength ($p_0 = 0$ and $\vec{p} \to 0$) limit, the time-component and spatial-component of $\Pi_V^{\mu\nu}(P)$ give the baryon number susceptibility ξ_n and baryon current susceptibility ξ_c, respectively,

$$
\xi_n \equiv -\lim_{\vec{p}\to 0}\tilde{\Pi}_V^{00}(0, \vec{p}) \propto m_{8,D}^2, \tag{15}
$$

$$
\xi_c \equiv -\frac{1}{2}\lim_{\vec{p}\to 0}\left(g_{ij} + \frac{p_i p_j}{p^2} \right)\Pi_V^{ij}(0, \vec{p}) \propto m_{8,M}^2. \tag{16}
$$

In the g2SC phase, $m_{8,M}^2$ as well as ξ_c become negative. This means that, except the chromomagnetic instability corresponding to broken generators of $SU(3)_c$, and the instability of a net momentum for diquark pair, the g2SC phase is also unstable with respect to an external color neutral baryon current $\bar{\psi}\vec{\gamma}\psi$.

The 8-th gluon's magnetic instability, the diquark momentum instability and the color neutral baryon current in the g2SC phase can be understood in one common physical picture. The g2SC phase exhibits a paramagnetic response to an external baryon current. Naturally, the color current carried by the 8-th gluon, which differs from the baryon current by a color charge, also experiences the instability in the g2SC phase. The paramagnetic instability of the baryon current indicates that the quark can spontaneously obtain a momentum, because diquark carries twice of the quark momentum, it is not hard to understand why the g2SC phase is also unstable with respect to the response of a net diquark momentum.

IV. SPONTANEOUS BARYON CURRENT GENERATION AND THE LOFF STATE

A. Spontaneous baryon current generation

The paramagnetic response to an external vector current naturally suggests that a vector current can be spontaneously generated in the system. The generated vector current behaves as a vector potential, which modifies the quark self-energy with a spatial vector condensate $\vec{\gamma} \cdot \vec{\Sigma}_V$, and breaks the rotational symmetry of the system. It can also be understood that the quasiparticles in the gapless phase spontaneously obtain a superfluid velocity, and the ground state is in an anisotropic state. The quark propagator $G_0^{\pm}(P)$ in Eq. (8) is modified as

$$\left[G_{0,V}^{\pm}(P) \right]^{-1} = \gamma^0 (p_0 \pm \hat{\mu}) - \vec{\gamma} \cdot \vec{p} \mp \vec{\gamma} \cdot \vec{\Sigma}_V, \quad (17)$$

with a subscript V indicating the modified quark propagator. Correspondingly, the inverse of the quark propagator $[\mathcal{S}(P)]^{-1}$ in Eq. (6) is modified as

$$[\mathcal{S}_V(P)]^{-1} = \begin{pmatrix} \left[G_{0,V}^{+}(P) \right]^{-1} & \Delta^- \\ \Delta^+ & \left[G_{0,V}^{-}(P) \right]^{-1} \end{pmatrix}. \quad (18)$$

It is noticed that the expression of the modified inverse quark propagator $[S_V(P)]^{-1}$ takes the same form as the inverse quark propagator in the one-plane wave LOFF state shown in Ref. [29]. The net momentum $\vec{q}$ of the diquark pair in the LOFF state [29] is replaced here by a spatial vector condensate $\vec{\Sigma}_V$. The spatial vector condensate $\vec{\gamma} \cdot \vec{\Sigma}_V$ breaks rotational symmetry of the system. This means that the Fermi surfaces of the pairing quarks are not spherical any more.

It has to be pointed out, the baryon current offers one Doppler-shift superfluid velocity for the quarks. A spontaneously generated Nambu-Goldstone current in the minimal gapless model [41] or a condensate of 8-th gluon's spatial component can do the same job. All these states mimic the one-plane wave LOFF state. In the following, we just call all these states as the single-plane wave LOFF state.

In order to determine the deformed structure of the Fermi surfaces, one should self-consistently minimize the free energy $\Gamma(\Sigma_V, \Delta, \mu, \mu_e, \mu_8)$. The explicit form of the free energy can be evaluated directly using the standard method, in the framework of Nambu–Jona-Lasinio model [8, 13], it takes the form of

$$\Gamma = -\frac{T}{2} \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \text{Tr} \ln([\mathcal{S}_V(P)]^{-1}) + \frac{\Delta^2}{4G_D}, \quad (19)$$

where T is the temperature, and G_D is the coupling constant in the diquark channel.

When there is no charge neutrality condition, the ground state is determined by the thermal stability condition, i.e., the local stability condition. The ground state

is in the 2SC phase when $\delta\mu < 0.706\Delta_0$ with $\Delta \simeq \Delta_0$, in the LOFF phase when $0.706\Delta_0 < \delta\mu < 0.754\Delta_0$ correspondingly $0 < \Delta/\Delta_0 < 0.242$, and then in the normal phase with $\Delta = 0$ when the mismatch is larger than $0.754\Delta_0$. Here Δ, Δ_0 indicate the diquark gap in the case of $\delta\mu \neq 0$ and $\delta\mu = 0$, respectively.

B. Unstable neutral LOFF window

Now come to the charge neutral LOFF state, and investigate whether the LOFF state can resolve all the magnetic instabilities.

When charge neutrality condition is required, the ground state of charge neutral quark matter should be determined by solving the gap equations as well as the charge neutrality condition, i.e.,

$$\frac{\partial\Gamma}{\partial\Sigma_V} = 0, \quad \frac{\partial\Gamma}{\partial\Delta} = 0, \quad \frac{\partial\Gamma}{\partial\mu_e} = 0, \quad \frac{\partial\Gamma}{\partial\mu_8} = 0. \quad (20)$$

By changing Δ_0 or coupling strength G_D, the solution of the charge neutral LOFF state can stay everywhere in the full LOFF window, including the window not protected by the local stability condition, as shown explicitly in Ref. [31].

From the lesson of charge neutral g2SC phase, we learn that even though the neutral state is a thermal stable state, i.e., the thermodynamic potential is a global minimum along the neutrality line, it cannot guarantee the dynamical stability of the system. The stability of the neutral system should be further determined by the dynamical stability condition, i.e., the positivity of the Meissner mass square.

The polarization tensor for the gluons with color $A = 4, 5, 6, 7, 8$ should be evaluated using the modified quark propagator $\mathcal{S}_V$ in Eq. (18), i.e.,

$$\Pi_{AB}^{\mu\nu}(P) = \frac{1}{2} \frac{T}{V} \sum_K \text{Tr} \left[\hat{\Gamma}_A^\mu \mathcal{S}_V(K) \hat{\Gamma}_B^\nu \mathcal{S}_V(K - P) \right], \quad (21)$$

with $A, B = 4, 5, 6, 7, 8$ and the explicit form of the vertices $\hat{\Gamma}_A^\mu$ has the form $\hat{\Gamma}_A^\mu \equiv \text{diag}(g\,\gamma^\mu T_A, -g\,\gamma^\mu T_A^T)$. In the LOFF state, the Meissner tensor can be decomposed into transverse and longitudinal component. The transverse and longitudinal Meissner mass square for the off-diagonal 4-7 gluons and the diagonal 8-th gluon have been performed explicitly in the one-plane wave LOFF state in Ref. [29].

According to the dynamical stability condition, i.e., the positivity of the transverse as well as longitudinal Meissner mass square, we can devide the LOFF state into three LOFF windows [34]:

1) The stable LOFF (S-LOFF) window in the region of $0 < \Delta/\Delta_0 < 0.39$, which is free of any magnetic instability. Please note that this S-LOFF window is a little bit wider than the window $0 < \Delta/\Delta_0 < 0.242$ protected by the local stability condition.

2) The stable window for diagonal gluon characterized by Ds-LOFF window in the region of $0.39 < \Delta/\Delta_0 < 0.83$, which is free of the diagonal 8-th gluon's magnetic instability but not free of the off-diagonal gluons' magnetic instability;

3) The unstable LOFF (Us-LOFF) window in the region of $0.83 < \Delta/\Delta_0 < r_c$, with $r_c \equiv \Delta(\delta\mu = \Delta)/\Delta_0 \simeq 1$. In this Us-LOFF window, all the magnetic instabilities exist. Please note that, it is the longitudinal Meissner mass square for the 8-th gluon is negative in this Us-LOFF window, the transverse Meissner mass square of 8-th gluon is always zero in the full LOFF window, which is guaranteed by the momentum equation.

Us-LOFF is a very interesting window, it indicates that the LOFF state even cannot cure the 8-th gluon's magnetic instability. In the charge neutral 2-flavor system, it seems that the diagonal gluon's magnetic instability cannot be cured in the gluon phase, because there is no direct relation between the diagonal gluon's instability and the off-diagonal gluons' instability. (Of course, it has to be carefully checked, whether all the instabilities in this Us-LOFF window can be cured by off-diagonal gluons' condensate in the charge neutral 2-flavor system.) It is also noticed that in this Us-LOFF window, the mismatch is close to the diquark gap, i.e., $\delta\mu \simeq \Delta$. Therefore it is interesting to check whether this Us-LOFF window can be stabilized by a spin-1 condensate [35] as proposed in Ref. [41].

In the charge neutral 2SC phase, though it is unlikely, we might have a lucky chance to cure the diagonal instability by the condensation of off-diagonal gluons. However, this instability still exists in some constrained Abelian asymmetric superfluidity system, and it's a new challenge for us to really solve this problem.

V. SPONTANEOUS NAMBU-GOLDSTONE CURRENTS GENERATION

We have seen that, except chromomagnetic instability corresponding to broken generators of $SU(3)_c$, the g2SC phase is also unstable with respect to the external neutral baryon current. It is noticed that all the instabilities are induced by increasing the mismatch between the Fermi surfaces of the Cooper pairing. In order to understand the instability driven by mismatch, in the following, we give some general analysis.

A superconductor will be eventually destroyed and goes to the normal Fermi liquid state, so one natural question is: how an ideal BCS superconductor will be destroyed by increasing mismatch? To answer how a superconductor will be destroyed, one has to firstly understand what is a superconductor. The superconducting phase is characterized by the order parameter $\Delta(x)$, which is a complex scalar field and has the form of e.g., for electrical superconductor, $\Delta(x) = |\Delta|e^{i\varphi(x)}$, with $|\Delta|$ the amplitude and φ the phase of the gap order parameter or the pseudo Nambu-Goldstone boson.

1) The superconducting phase is charaterized by the nonzero vacuum expectation value, i.e., $< \Delta > \neq 0$, which means the amplitude of the gap is finite, and the phase coherence is also established.

2) If the amplitude is still finite, while the phase coherence is lost, this phase is in a phase decoherent pseudogap state characterized by $|\Delta| \neq 0$, but $< \Delta >= 0$ because of $< e^{i\varphi(x)} >= 0$.

3) The normal state is characterized by $|\Delta| = 0$.

There are two ways to destroy a superconductor. One way is by driving the amplitude of the order parameter to zero. This way is BCS-like, because it mimics the behavior of a conventional superconductor at finite temperature, the gap amplitude monotonously drops to zero with the increase of temperature; Another way is non-BCS like, but Berezinskii-Kosterlitz-Thouless (BKT)-like [36], even if the amplitude of the order parameter is large and finite, superconductivity will be lost with the destruction of phase coherence, e.g. the phase transition from the $d-$wave superconductor to the pseudogap state in high temperature superconductors [37].

Stimulating by the role of the phase fluctuation in the unconventional superconducting phase in condensed matter, we follow Ref. [38] to formulate the 2SC phase in the nonlinear realization framework in order to naturally take into account the contribution from the phase fluctuation or pseudo Nambu-Goldstone current.

In the 2SC phase, the color symmetry $G = SU(3)_c$ breaks to $H = SU(2)_c$. The generators of the residual $SU(2)_c$ symmetry H are $\{S^a = T^a\}$ with $a = 1, 2, 3$ and the broken generators $\{X^b = T^{b+3}\}$ with $b = 1, \cdots, 5$. More precisely, the last broken generator is a combination of T_8 and the generator $\mathbf{1}$ of the global $U(1)$ symmetry of baryon number conservation, $B \equiv (\mathbf{1} + \sqrt{3}T_8)/3$ of generators of the global $U(1)_B$ and local $SU(3)_c$ symmetry.

The coset space G/H is parameterized by the group elements

$$\mathcal{V}(x) \equiv \exp\left[i\left(\sum_{a=4}^{7} \varphi_a(x)T_a + \frac{1}{\sqrt{3}}\varphi_8(x)B \right) \right] , \quad (22)$$

here $\varphi_a(a = 4, \cdots, 7)$ and φ_8 are five Nambu-Goldstone diquarks, and we have neglected the singular phase, which should include the information of the topological defects [39, 40]. Operator $\mathcal{V}$ is unitary, $\mathcal{V}^{-1} = \mathcal{V}^\dagger$.

Introducing a new quark field χ, which is connected with the original quark field q in Eq. (3) in a nonlinear transformation form,

$$q = \mathcal{V}\chi , \quad \bar{q} = \bar{\chi}\mathcal{V}^\dagger , \quad (23)$$

and the charge-conjugate fields transform as

$$q_C = \mathcal{V}^*\chi_C , \quad \bar{q}_C = \bar{\chi}_C\mathcal{V}^T . \quad (24)$$

In high-T_c superconductor, this technique is called charge-spin separation, see Ref. [39]. The advantage of transforming the quark fields is that this preserves the

simple structure of the terms coupling the quark fields to the diquark sources,

$$\bar{q}_C \, \Delta^+ q \equiv \bar{\chi}_C \, \Phi^+ \chi \;\; , \;\; \bar{q} \, \Delta^- q_C \equiv \bar{\chi} \, \Phi^- \chi_C \; . \quad (25)$$

In mean-field approximation, the diquark source terms are proportional to

$$\Phi^+ \sim \langle \, \chi_C \, \bar{\chi} \, \rangle \;\; , \;\; \Phi^- \sim \langle \, \chi \, \bar{\chi}_C \, \rangle \; . \quad (26)$$

Introducing the new Nambu-Gor'kov spinors

$$X \equiv \begin{pmatrix} \chi \\ \chi_C \end{pmatrix} \;\; , \;\; \bar{X} \equiv (\bar{\chi} \, , \, \bar{\chi}_C), \quad (27)$$

the nonlinear realization of the original Lagrangian density Eq.(3) takes the form of

$$\mathcal{L}_{nl} \equiv \bar{X} \, \mathcal{S}_{nl}^{-1} \, X - \frac{\Phi^+ \Phi^-}{4G_D} \; , \quad (28)$$

where

$$\mathcal{S}_{nl}^{-1} \equiv \begin{pmatrix} [G_{0,nl}^+]^{-1} & \Phi^- \\ \Phi^+ & [G_{0,nl}^-]^{-1} \end{pmatrix} \; . \quad (29)$$

Here the explicit form of the free propagator for the new quark field is

$$[G_{0,nl}^+]^{-1} \; = \; i \, \slashed{D} + \hat{\mu} \, \gamma_0 + \gamma_\mu V^\mu, \quad (30)$$

and

$$[G_{0,nl}^-]^{-1} \; = \; i \, \slashed{D}^T - \hat{\mu} \, \gamma_0 + \gamma_\mu V_C^\mu. \quad (31)$$

Comparing with the free propagator in the original Lagrangian density, the free propagator in the non-linear realization framework naturally takes into account the contribution from the Nambu-Goldstone currents or phase fluctuations, i.e.,

$$V^\mu \; \equiv \; \mathcal{V}^\dagger \, (i \, \partial^\mu) \, \mathcal{V}, \\ V_C^\mu \; \equiv \; \mathcal{V}^T \, (i \, \partial^\mu) \, \mathcal{V}^*, \quad (32)$$

which is the $N_c N_f \times N_c N_f$-dimensional Maurer-Cartan one-form introduced in Ref. [38]. The linear order of the Nambu-Goldstone currents V^μ and V_C^μ has the explicit form of

$$V^\mu \; \simeq \; -\sum_{a=4}^7 (\partial^\mu \varphi_a) \, T_a - \frac{1}{\sqrt{3}} \, (\partial^\mu \varphi_8) \, B \; , \quad (33)$$

$$V_C^\mu \; \simeq \; \sum_{a=4}^7 (\partial^\mu \varphi_a) \, T_a^T + \frac{1}{\sqrt{3}} \, (\partial^\mu \varphi_8) \, B^T \; . \quad (34)$$

The Lagrangian density Eq. (28) for the new quark fields looks like an extension of the theory in Ref. [39] for high-T_c superconductor to Non-Abelian system, except that here we neglected the singular phase contribution

from the topologic defects. The advantage of the non-linear realization framework Eq. (28) is that it can naturally take into account the contribution from the phase fluctuations or Nambu-Goldstone currents.

The task left is to correctly solve the ground state by considering the phase fluctuations. The free energy $\Gamma(V_\mu, \Delta, \mu, \mu_8, \mu_e)$ can be evaluated directly and it takes the form of

$$\Gamma = -\frac{T}{2} \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \text{Tr} \ln([\mathcal{S}_{nl}(P)]^{-1}) + \frac{\Phi^2}{4G_D}. \quad (35)$$

To evaluate the ground state of $\Gamma(V_\mu, \Delta, \mu, \mu_8, \mu_e)$ as a function of mismatch is tedious and still under progress. In the following we just give a brief discussion on the Nambu-Goldstone current generation state [41], one-plane wave LOFF state [28, 29], as well as the gluon phase [32].

If we expand the thermodynamic potential $\Gamma(V_\mu, \Delta, \mu, \mu_8, \mu_e)$ of the non-linear realization form in terms of the Nambu-Goldstone currents, we will naturally have the Nambu-Goldstone currents generation in the system with the increase of mismatch, i.e., $< \sum_{a=4}^7 \vec{\nabla}\varphi_a >\neq 0$ and/or $< \vec{\nabla}\varphi_8 >\neq 0$ at large $\delta\mu$. This is an extended version of the Nambu-Goldstone current generation state proposed in a minimal gapless model in Ref. [41, 42]. From Eq. (28), we can see that $\vec{\nabla}\varphi_8$ contributes to the baryon current. $< \vec{\nabla}\varphi_8 >\neq 0$ indicates a baryon current generation or 8-th gluon condensate in the system, it is just the one-plane wave LOFF state. This has been discussed in Sec. IV. The other four Nambu-Goldstone currents generation $< \sum_{a=4}^7 \vec{\nabla}\varphi_a >\neq 0$ indicates other color current generation in the system, and is equivalent to the gluon phase described in Ref. [32].

We do not argue whether the system will exprience a gluon condensate phase or Nambu-Goldstone currents generation state. We simply think they are equivalent. In fact, the gauge fields and the Nambu-Goldstone currents share a gauge covariant form as shown in the free propogator. However, we prefer to using Nambu-Goldstone currents generation than the gluon condensate in the gNJL model. As mentioned in Sec. II, in the gNJL model, all the information from unkown nonperturbative gluons are hidden in the diquark gap parameter Δ. The gauge fields in the Lagrangian density are just external fields, they only play the role of probing the system, but do not contribute to the property of the color superconducting phase. Therefore, there is no gluon free-energy in the gNJL model, it is not clear how to derive the gluon condensate in this model. In order to investigate the problem in a fully self-consistent way, one has to use the ambitious framework by using the Dyson-Schwinger equations (DSE) [43] including diquark degree of freedom [44] or in the framework of effective theory of high-density quark matter as in Ref. [45].

VI. CONCLUSION AND DISCUSSION

In this paper, we show that, except the chromomagnetic instability, the g2SC phase also exhibits a paramagnetic response to the perturbation of an external baryon current. This suggests a baryon current can be spontaneously generated in the g2SC phase, and the quasiparticles spontaneously obtain a superfluid velocity. The spontaneously generated baryon current breaks the rotational symmetry of the system, and it is equivalent to the one-plane wave LOFF state.

We further describe the 2SC phase in the nonlinear realization framework, and show that each instability indicates the spontaneous generation of the corresponding pseudo Nambu-Goldstone current. We show this Nambu-Goldstone currents generation state can naturally cover the gluon phase as well as the one-plane wave LOFF state.

We also point out that, when charge neutrality condition is required, there exists a narrow unstable LOFF (Us-LOFF) window, where not only off-diagonal gluons but the diagonal 8-th gluon cannot avoid the magnetic instability. The diagonal gluon's magnetic instability in this Us-LOFF window cannot be cured by off-diagonal gluon condensate in color superconducting phase. More interestingly, this Us-LOFF window will also show up in some constrained Abelian asymmetric superfluid system.

The Us-LOFF window brings us a new challenge. We need new thoughts on understanding how a BCS supercondutor will be eventually destroyed by increasing the mismatch, we also need to develop new methods to really resolve the instability problem. Some methods developed in unconventional superconductor field, e.g., High-T_c superconductor, might be helpful. The work toward this direction is in progressing.

In this paper, we did not discuss the magnetic instabilty in the gCFL phase. After the first version of this paper appeared, the author was informed by D. T. Son that the baryon current generation was also found in the gCFL phase [46]. For more discussion on solving the magnetic instabilty in the gCFL phase, please refer to Ref. [47].

Acknowledgments

[1] R. Rapp, T. Schäfer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998); M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422**, 247 (1998).

[2] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).

[3] D. T. Son, Phys. Rev. D **59**, 094019 (1999); D. K. Hong, V. A. Miransky, I. A. Shovkovy, and L. C. R. Wijewardhana, Phys. Rev. D **61**, 056001 (2000); D. K. Hong, Nucl. Phys. **B582**, 451 (2000); T. Schäfer and F. Wilczek, Phys. Rev. D **60**, 114033 (1999); R.D. Pisarski and D.H. Rischke, Phys. Rev. D **61**, 051501 (2000); S. D. H. Hsu and M. Schwetz, Nucl. Phys. **B572**, 211 (2000); W. E. Brown, J. T. Liu, and H.-C. Ren, Phys. Rev. D **61**, 114012 (2000).

[4] T. Schäfer, Nucl. Phys. **B575**, 269 (2000); I. A. Shovkovy and L. C. R. Wijewardhana, Phys. Lett. B **470**, 189 (1999).

[5] K. Rajagopal and F. Wilczek, hep-ph/0011333; D. K. Hong, Acta Phys. Polon. B **32**, 1253 (2001); M. Alford, Ann. Rev. Nucl. Part. Sci. **51**, 131 (2001); T. Schäfer, hep-ph/0304281; D. H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004); M. Buballa, hep-ph/0402234; H.-C. Ren, hep-ph/0404074; M. Huang, hep-ph/0409167; I. A. Shovkovy, nucl-th/0410091.

[6] M. Alford and K. Rajagopal, JHEP **0206**, 031 (2002).

[7] A.W. Steiner, S. Reddy and M. Prakash, Phys. Rev. D **66**, 094007 (2002).

[8] M. Huang, P. F. Zhuang and W. Q. Chao, Phys. Rev. D **67**, 065015 (2003).

[9] P. F. Bedaque, Nucl. Phys. A **697**, 569 (2002); O. Kiriyama, S. Yasui and H. Toki, Int. J. Mod. Phys. E **10**, 501 (2001).

[10] A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965); P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).

[11] M. G. Alford, J. A. Bowers and K. Rajagopal, Phys. Rev. D **63**, 074016 (2001); D. K. Hong and Y. J. Sohn, hep-ph/0107003; I. Giannakis, J. T. Liu and H. c. Ren, Phys. Rev. D **66**, 031501 (2002); R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. **76**, 263 (2004); J. A. Bowers, hep-ph/0305301.

[12] M. G. Alford, K. Rajagopal, S. Reddy and F. Wilczek, Phys. Rev. D **64**, 074017 (2001) F. Neumann, M. Buballa, and M. Oertel, Nucl. Phys. **A714**, 481 (2003); I. Shovkovy, M. Hanauske and M. Huang, Phys. Rev. D **67**, 103004 (2003); H. Grigorian, D. Blaschke and D. N. Aguilera, Phys. Rev. C **69**, 065802 (2004); S. Reddy and G. Rupak, Phys. Rev. C **71**, 025201 (2005).

[13] I. Shovkovy and M. Huang, Phys. Lett. B **564**, 205 (2003).

[14] M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. Lett. **92**, 222001 (2004); Phys. Rev. D **71**, 054009 (2005).

[15] G. Sarma, J. Phys. Chem. Solids **24**, 1029 (1963).

[16] M. G. Alford, J. Berges and K. Rajagopal, Phys. Rev. Lett. **84**, 598 (2000);

[17] E. Gubankova, W. V. Liu and F. Wilczek, Phys. Rev. Lett. **91**, 032001 (2003); A. Mishra and H. Mishra, Phys. Rev. D **69**, 014014 (2004); S. B. Rüster and

D. H. Rischke, Phys. Rev. D **69**, 045011 (2004).

[18] M. Huang and I. Shovkovy, Nucl. Phys. **A729**, 835 (2003); J. F. Liao and P. F. Zhuang, Phys. Rev. D **68**, 114016 (2003). K. Fukushima, C. Kouvaris and K. Rajagopal, Phys. Rev. D **71**, 034002 (2005).

[19] S. B. Rüster, I. A. Shovkovy and D. H. Rischke, Nucl. Phys. **A743**, 127 (2004); H. Abuki, M. Kitazawa and T. Kunihiro, hep-ph/0412382; S. B. Ruster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, hep-ph/0503184; D. Blaschke, S. Fredriksson, H. Grigorian, A. M. Oztas and F. Sandin, hep-ph/0503194.

[20] W. Meissner and R. Ochsenfeld, Naturwiss. **21**, 787 (1933).

[21] D. H. Rischke, Phys. Rev. D **62**, 034007 (2000); D. H. Rischke, Phys. Rev. D **62**, 054017 (2000).

[22] M. Huang and I. A. Shovkovy, Phys. Rev. D **70**, 051501 (2004).

[23] M. Huang and I. A. Shovkovy, Phys. Rev. D **70**, 094030 (2004).

[24] R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli and M. Ruggieri, Phys. Lett. B **605**, 362 (2005); M. Alford and Q. h. Wang, hep-ph/0501078.

[25] S.-T. Wu and S. Yip, Phys. Rev. A **67**, 053603 (2003).

[26] W. V. Liu and F. Wilczek, Phys. Rev. Lett. **90**, 047002 (2003).

[27] K. Iida, T. Matsuura, M. Tachibana and T. Hatsuda, Phys. Rev. D **71**, 054003 (2005).

[28] I. Giannakis and H. C. Ren, Phys. Lett. B **611**, 137 (2005).

[29] I. Giannakis and H. C. Ren, Nucl. Phys. B **723**, 255 (2005).

[30] I. Giannakis, D. f. Hou and H. C. Ren, Phys. Lett. B **631**, 16 (2005)

[31] E. V. Gorbar, M. Hashimoto and V. A. Miransky, hep-ph/0509334.

[32] E. V. Gorbar, M. Hashimoto and V. A. Miransky, hep-ph/0507303.

[33] M. Huang, P. f. Zhuang and W. q. Chao, Phys. Rev. D **65**, 076012 (2002)

[34] We thank H.C. Ren for helpful discussion on this point.

[35] T. Schafer, Phys. Rev. D **62**, 094007 (2000); A. Schmitt, Q. Wang and D. H. Rischke, Phys. Rev. Lett. **91**, 242301 (2003); A. Schmitt, nucl-th/0405076.

[36] V.L.Berezinskii, Zh. Eksp. Theor. Fiz. **59**, 907(1970) [Sov. Phys. JETP **32**, 493(1971)]; J.M.Kosterlitz and D.J. Thouless, J. Phys. C**6**, 1181 (1973).

[37] V.J. Emery and S.A. Kivelson, Nature (London) **374**, 434(1995).

[38] R. Casalbuoni, Z. y. Duan and F. Sannino, Phys. Rev. D **62**, 094004 (2000); D. H. Rischke and I. A. Shovkovy, Phys. Rev. D **66**, 054019 (2002)

[39] M.Franz, Z. Tesanovic, Phys.Rev.Lett.87, 257003(2001); M.Franz, Z. Tesanovic, O.Vafek, Phys.Rev.**B66**(2002), 054535; A. Melikyan, Z. Tesanovic, cond-mat/0408344.

[40] H. Kleinert, cond-mat/9503030.

[41] D. K. Hong, hep-ph/0506097.

[42] M. Huang, hep-ph/0509177.

[43] P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297 (2003); C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000)

[44] D. K. Hong, V. A. Miransky, I. A. Shovkovy and L. C. R. Wijewardhana, Phys. Rev. D **61**, 056001 (2000), [Erratum-ibid. D **62**, 059903 (2000)]; D. D. Dietrich and D. H. Rischke, Prog. Part. Nucl. Phys. **53**, 305 (2004).

[45] P. T. Reuter, Q. Wang and D. H. Rischke, Phys. Rev. D **70**, 114029 (2004), [Erratum-ibid. D **71**, 099901 (2005)].

[46] private communication.

[47] A. Kryjevski, hep-ph/0508180; T. Schafer, hep-ph/0508190; K. Fukushima, hep-ph/0510299.